

Fast control of topological vortex formation in BEC by counter-diabatic driving

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Topological vortex formation has been known as the simplest method for vortex formation in BEC of alkali atoms. This scheme requires inversion of the bias magnetic field along the axis of the condensate, which leads to atom loss when the bias field crosses zero. In this Letter, we propose a scheme with which the atom loss is greatly suppressed by adding counter-diabatic magnetic field. A naive counter-diabatic field violates the Maxwell equations and we need to introduce an approximation to make it physically feasible. The resulting field requires an extra currents, which is experimentally challenging. Finally we solve this problem by applying a gauge transformation so that the counter-diabatic field is generated by controlling the original trap field with the additional control of the bias field.

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Demonstration of quantized vortices in a superfluid is a manifestation of the nonvanishing order parameter. Topological structure of vortices often reflects the manifold of the order parameter space. It is, therefore, natural to seek for the method to experimentally demonstrate formation of vortices once BEC of alkali metal atoms has been realized. Topological method of vortex formation is the simplest one compared with other proposals, where the spinor structure of the order parameter is fully utilized [1–7]. Although the vortex [6] and monopole [8, 9] in BEC have been observed experimentally, establishment of methods to create such interesting quantum states more accurately in time much shorter than coherence time is important for further experimental study and applications.

A condensate in the weak field seeking state (WFSS) is trapped in a quadrupole magnetic field in the xy -plane with a uniform bias field along the z -axis first and, subsequently, the bias field is reversed from B_z to $-B_z$ adiabatically. Then a vortex with the winding number $2F$ is formed, where F is the quantum number of the hyperfine spin of the condensate. This theoretical proposal has been demonstrated experimentally by several groups [10–15]. A part of the condensate is lost while vortex formation takes place. This is due to the fact that the gap between the WFSS and the neutral state (NS) and the strong field seeking state (SFSS) is gradually closed near the center of the condensate during the vortex formation and the atoms in the trapping WFSS make transition to the NS and the SFSS through the Majorana flops. It is the purpose of this Letter to show that it is possible to suppress the Majorana flops so that more atoms are left in the trap when a vortex is formed. It is certainly desirable to have more atoms left in the trap since higher atomic density implies (i) easier measurement of

physical quantities (ii) fighting against atom loss due to hyperfine spin changing collision and (iii) smaller vortex core size, which is advantageous for forming a vortex in an arbitrary position in the condensate [16]. Motivated by the concept of shortcuts to adiabaticity [17], we use the counter-diabatic formalism (or quantum transitionless algorithm) developed by Demirplak and Rice [18] and formulated by Berry [19, 20] and demonstrated experimentally in [21]. The supplementary counter-diabatic field (CDF) thus obtained, is, however, unphysical in that it violates the Maxwell equation $\text{div } \mathbf{B} = 0$ and some approximation must be introduced so that the counter-diabatic control is physically feasible. The approximated CDF turns out to be a quadrupole field, which can be implemented with an ordinary Ioffe bars. However, the CDF is generated by another set of Ioffe bars, which is obtained by rotating the original confining Ioffe bars by $\pi/4$. This means that two sets of Ioffe bars are required to physically realize this nonadiabatic control, which makes its experimental demonstration rather demanding. To overcome this difficulty, we apply the gauge transformation [22] to make the combined magnetic field (the original confining field and the CDF superposed) parallel to the original confining quadrupole field. The modest price one has to pay by this gauge transformation is to modify the time-dependence of the bias field $B_z(t)$ from a linear behavior. Experimental demonstration is made much easier with this modification.

We first regard each atom with hyperfine \mathbf{F} as an independent quantum system fixed at \mathbf{r} and consider nonadiabatic control thereof. Let $\mathbf{B}(\mathbf{r}, t) = (\mathbf{B}_\perp(\mathbf{r}), B_z(t))$ be the local magnetic field with the quadrupole field $\mathbf{B}_\perp(\mathbf{r}) = B'_\perp(x, -y)$ and the uniform bias field $B_z(t) = B_z(1 - 2t/T)$ for $0 \leq t \leq T$, $B_z(t) = -B_z(0)$ for $t > T$ where T is the inversion time of $B_z(t)$. An atom at (\mathbf{r}, t)

interacts with the magnetic field through the Hamiltonian

$$H_B(\mathbf{r}, t) = \gamma \mathbf{B}(\mathbf{r}, t) \cdot \mathbf{F}, \quad (1)$$

where $\gamma = \mu_B g_F$, μ_B is the Bohr magneton, g_F is the g -factor for the hyperfine spin F and $\mathbf{F} = (F_x, F_y, F_z)$ is $(2F + 1)$ -dimensional irreducible representation of the Lie algebra $\mathfrak{su}(2)$. We take $F = 1$ in the following for definiteness. For a given $\mathbf{B}(\mathbf{r}, t)$, the Hamiltonian (1) has three normalized eigenstates

$$\begin{aligned} |\text{WFSS}\rangle &= \frac{1}{2B} \begin{pmatrix} (B - B_z)e^{2i\phi} \\ -\sqrt{2}B_\perp e^{i\phi} \\ B + B_z \end{pmatrix}, \\ |\text{NS}\rangle &= \frac{1}{\sqrt{2}B} \begin{pmatrix} -B_\perp e^{2i\phi} \\ \sqrt{2}B_z e^{i\phi} \\ B_\perp \end{pmatrix}, \\ |\text{SFSS}\rangle &= \frac{1}{2B} \begin{pmatrix} (B + B_z)e^{2i\phi} \\ \sqrt{2}B_\perp e^{i\phi} \\ B - B_z \end{pmatrix}, \end{aligned} \quad (2)$$

where $B = \sqrt{B_\perp^2(r) + B_z^2(t)}$ and ϕ is the azimuthal angle. The corresponding eigenvalues are $|\gamma|B$, 0, and $-|\gamma|B$, respectively, where $\gamma = -\mu_B/2$. The WFSS with the highest energy $|\gamma|B$ is the only state that can be magnetically trapped. We start with a pure WFSS at $t = 0$ and want to keep as many atoms in the WFSS as possible at $t = T$, when the vortex formation is complete, in spite of vanishing gaps among the three states at the origin ($r = 0$) at $t = T/2$. This is realized by adding a CDF on the atom.

Here we briefly summarize the counter-diabatic approach to nonadiabatic quantum control and apply it to our hyperfine spin control. Let $H_0(t)$ be a time-dependent Hamiltonian and $|n(t)\rangle$ be an instantaneous eigenvector with the eigenvalue E_n such that $H_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$. In the adiabatic approximation, the solution to the time dependent Schrödinger equation is $|\psi_n(t)\rangle = e^{-i\gamma_n(t)}|n(t)\rangle$, where

$$\gamma_n(t) = \frac{1}{\hbar} \int_0^t dt' E_n(t') - i \int_0^t dt' \langle n(t') | \partial_{t'} n(t') \rangle. \quad (3)$$

Let $U(t) = \sum_n |\psi_n(t)\rangle \langle n(0)|$ be the time-evolution operator such that $U(t) : |n(0)\rangle \mapsto |\psi_n(t)\rangle$. The operator $U(t)$ defines a Hamiltonian $H(t) = i\hbar(\partial_t U(t))U^\dagger(t)$ for an arbitrary time evolution, not necessarily adiabatic, namely $|\psi_n(t)\rangle$ is the *exact* solution of

$$i\hbar\partial_t |\psi_n(t)\rangle = H(t)|\psi_n(t)\rangle, \quad (4)$$

where $|\psi_n(0)\rangle = |n(0)\rangle$ is satisfied by definition. If we write $H(t) = H_0(t) + H_{\text{CD}}(t)$, the counter-diabatic Hamiltonian $H_{\text{CD}}(t)$ is written as

$$H_{\text{CD}}(t) = i\hbar \sum_n |\partial_t n(t)\rangle \langle n(t)|, \quad (5)$$

when the “parallel” condition, $\langle n(t) | \partial_t n(t) \rangle = 0$, is satisfied.

Let us apply the counter-diabatic scheme to our hyperfine spin system by taking H_0 and $|n\rangle$ as $H_0(\mathbf{r}, t) = H_B(\mathbf{r}, t)$ and the instantaneous eigenstates in Eqs. (2). Note that the coordinate \mathbf{r} here is just a parameter specifying the position of the atom in the condensate, and the geometric phase in Eq. (3) vanishes. By substituting Eq. (2) into (5), the counter-diabatic Hamiltonian is obtained as

$$H_{\text{CD}} = \gamma \mathbf{B}_{\text{CD}} \cdot \mathbf{F}, \quad (6)$$

where

$$\mathbf{B}_{\text{CD}}(\mathbf{r}, t) = \frac{-2}{\gamma T} \frac{B_z(0)B'_\perp}{B^2(r, t)}(y, x, 0) \quad (7)$$

is the CDF. This magnetic field looks as if it could be generated by four Ioffe bars that are obtained by rotating the original Ioffe bars by an angle $\pi/4$ around the z -axis. An important remark is in order here. When the counter-diabatic scheme is applied to a quantum system, it produces a counter-diabatic potential (the magnetic field in our case) that prevents the quantum system from escaping from an adiabatic time evolution. There is no guarantee, however, that the counter-diabatic potential will be physically feasible. In fact, \mathbf{B}_{CD} in Eq. (7) does not satisfy $\text{div} \mathbf{B}_{\text{CD}} = 0$. This is not unexpected since we did not consider the Maxwell equation while we obtained \mathbf{B}_{CD} . The way out of this problem is to fix the coordinate r to r_0 in $B^2(r, t)$ in the denominator of $\mathbf{B}(r, t)$ so that $\mathbf{B}_{\text{CD}}(r, t) \propto (y, x, 0)$ and $\text{div} \mathbf{B}_{\text{CD}}$ vanishes. In our calculation below, we show that the CDF increases the number of atoms forming the vortex in a wide range of the inversion time T by using several values of r_0 . The performance may be further improved if we optimize the number of atoms left in the trap by varying r_0 , which will be worked out in our future publication. We call the combined magnetic field as $\tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}_{\text{CD}}$.

Now we solve the Gross-Pitaevskii equation (GPE) with the designed magnetic field $\tilde{\mathbf{B}}$;

$$\begin{aligned} i\hbar\partial_t \Psi_m(r, t) &= \left\{ h_{mn} + g_n \delta_{mn} \sum_p |\Psi_p|^2 \right. \\ &\quad \left. + g_s \sum_\alpha \sum_{l,p} (\Psi_l (F_\alpha)_{lp} \Psi_p) (F_\alpha)_{mn} \right\} \Psi_n, \end{aligned} \quad (8)$$

where $l, m, n, p \in \{-1, 0, 1\}$, $\alpha \in \{x, y, z\}$,

$$h_{mn} = \left(-\frac{\hbar^2 \nabla^2}{2M} - \mu \right) \delta_{mn} + \mathcal{B}_{mn}, \quad \mathcal{B} = \gamma \tilde{\mathbf{B}} \cdot \mathbf{F},$$

with the mass of the atom M , and the chemical potential μ (the eigenvalue of the GPE at $t = 0$). To begin with, we need to find the initial condition to solve the GPE (8). Assuming that the initial state is in the WFSS

$|\Psi(0)\rangle = f(\mathbf{r})|\text{WFSS}(0)\rangle$, the condensate wave function is obtained by solving the stationary GPE

$$-\frac{\hbar^2}{2M} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{\beta'^2}{2} - \frac{1}{4r^2} (7 - 8 \cos \beta + \cos 2\beta) \right] f(r) + \gamma \tilde{B}(\mathbf{r}, 0) f(r) + g f^3(r) = \mu f(r), \quad (9)$$

where $g = g_n + g_s$, $\tilde{B}(\mathbf{r}, t) = \{[\mathbf{B}_\perp(\mathbf{r}) + \mathbf{B}_{\text{CD}}(\mathbf{r}, t)]^2 + B_z^2(t)\}^{1/2}$, and the WFSS is parametrized as

$$|\text{WFSS}\rangle = \begin{pmatrix} \cos \beta \cos \phi - i \sin \phi \\ -\cos \beta \sin \phi - i \cos \phi \\ -\sin \beta \end{pmatrix},$$

with

$$\beta = \tan^{-1} \left[\frac{|\mathbf{B}_\perp(\mathbf{r}) + \mathbf{B}_{\text{CD}}(\mathbf{r}, t)|}{B_z(t)} \right].$$

Here we note that $f(\mathbf{r})$ and $|\mathbf{B}_\perp(\mathbf{r}) + \mathbf{B}_{\text{CD}}(\mathbf{r}, 0)|$ are in fact the functions of r only. It turns out that \mathbf{B}_{CD} is negligibly small at $t = 0$ and it can be ignored safely in solving Eq. (9). The time-dependent GPE is solved numerically and we summarize the results below. Throughout our calculation, we have taken the parameters of ^{23}Na atoms [3], $g_n = 0.0378 a_{\text{HO}}^3 \hbar \omega$, $g_s = 0$ and $B_z(0) = 1$ G, $B'_\perp = 300$ G/cm for which the harmonic oscillator length is found to be $a_{\text{HO}} \sim 9.14 \times 10^{-1} \mu\text{m}$ and $\hbar \omega \sim 3.49 \times 10^{-24}$ erg. Lengths and energies are scaled by these parameters. In addition, we find $\omega_L(r=0, t=0) \sim 4.40 \times 10^6$ rad/s. The time scale $\tau = 2\pi/\omega_L(0, 0) \sim 1.43 \mu\text{s}$ is a reasonable measure of adiabaticity. The chemical potential is found to be $\mu - \hbar \omega_L(0, 0) \sim 3.66$ in dimensionless units.

Figure 1 shows the time dependence of the ratio $|\mathbf{B}_{\text{CD}}|/|\mathbf{B}_\perp|$ for $\log_{10}(T/\tau) = 1.4$ and $r_0/a_{\text{HO}} = 2$, which is independent of r and depends only on t . The inset shows the profile of the condensate at $t = 0$, which is obtained by solving Eq. (9) numerically. The units of time, length and f are τ , a_{HO} and $a_{\text{HO}}^{-3/2}$, respectively. Now the GP equation (8) is solved to find interesting quantities. Let us show first how much atoms are left in the trap after the vortex formation takes place. To mimic the trap loss, we multiply the order parameter by $h(r) = 0.5[1 - \tanh((r - r_1)/\lambda)]$ with $r_1 = 30a_{\text{HO}}$ and $\lambda = 2a_{\text{HO}}$ at each time step of the numerical simulation [3]. In Fig. 2 we show the snapshots of the amplitude of the order parameter $\sum_n |\Psi_n|^2$ for $0 \leq t \leq 100T$ with the CDF with the same parameter set as Fig. 1. The condensate in the trapping region at $t = 100T$ is mostly in WFSS, while SFSS and NS are gradually removed from the trap in the domain $r > r_1$ due to $h(r)$. The zero of the order parameter at $r = 0$, $t = 100T$ is a manifestation of the vortex of the WFSS. Fig. 3 shows the fraction of atoms $N(t)/N(0)$ left in the trap at $t \gg T$. Since we replaced an unphysical \mathbf{B}_{CD} with a physical \mathbf{B}_{CD} satisfying $\text{div } \mathbf{B}_{\text{CD}} = 0$, the result depends on the parameter r_0

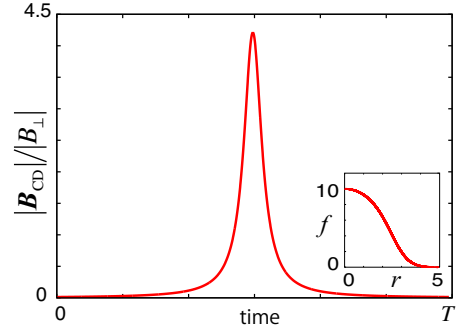


FIG. 1. (Color online) Time dependence of the ratio $|\mathbf{B}_{\text{CD}}|/|\mathbf{B}_\perp|$, which is independent of r and depends only on t . Inset shows the bound state wave function obtained by solving Eq. (9) for ^{23}Na .

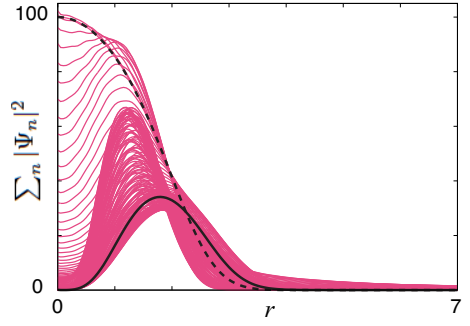


FIG. 2. (Color online) Snapshots of the amplitude of the order parameter for $0 \leq t \leq 100T$ with the same parameter set as that in Fig. 1. Time intervals between snapshots are equal with each other. The dashed and thick solid curves correspond to $t = 0$ and $t = 100T$, respectively. The unit of $\sum_n |\Psi_n|^2$ is a_{HO}^{-3} .

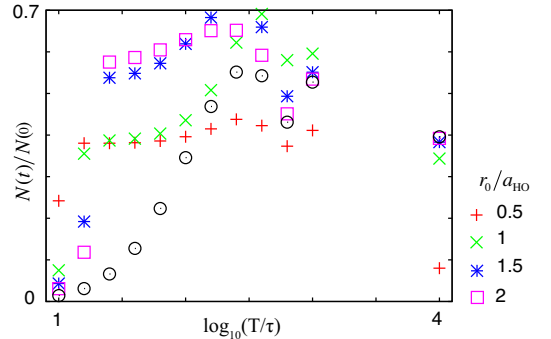


FIG. 3. (Color online) Fraction $N(t)/N(0)$ of atoms left in the trap long time after a vortex is formed. Since $t \gg T$, the condensate is in the pure WFSS. \circ shows the fraction without \mathbf{B}_{CD} while other symbols show the fractions with \mathbf{B}_{CD} for different r_0 .

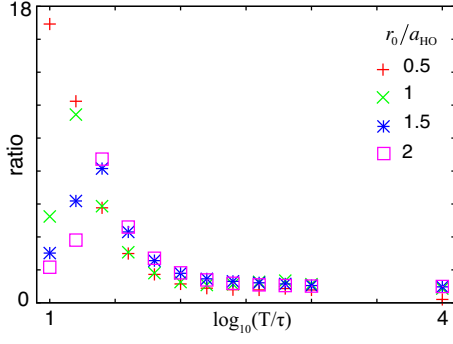


FIG. 4. (Color online) Number of atoms $N(t)$ for $t \gg T$ in the control with the CDF normalized by $N(t)$ in the control without the CDF. The ratio is larger for small T .

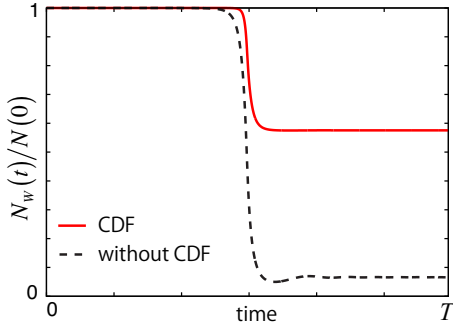


FIG. 5. (Color online) Instantaneous fraction $N_W(t)/N(0)$ of atoms in WFSS during vortex formation for $\log_{10}(T/\tau) = 1.4$ and $r_0 = 2a_{\text{HO}}$. The dashed and solid curves correspond to the controls without and with the CDF, respectively.

in the denominator of Eq. (7). Observe the prominent improvement in the ratio $N(t)/N(0)$ for small T . This is more clearly seen by plotting $N(t)$ in the control with the CDF normalized by $N(t)$ in the control without the CDF. Figure 4 shows the ratios with the same r_0 as Fig. 3. The ratio is almost 20 for $T = 10\tau$ and $r_0 = 0.5a_{\text{HO}}$.

Next, we show the CDF really suppresses the transitions $\text{WFSS} \rightarrow \text{NS}$ and $\text{WFSS} \rightarrow \text{SFSS}$ by looking at the projected atom numbers. For this purpose, we define the projection operators $\Pi_W = |\text{WFSS}\rangle\langle\text{WFSS}|$, $\Pi_N = |\text{NS}\rangle\langle\text{NS}|$ and $\Pi_S = |\text{SFSS}\rangle\langle\text{SFSS}|$ and evaluate the numbers of atoms in these states as

$$N_W = \langle\Psi|\Pi_W|\Psi\rangle, N_N = \langle\Psi|\Pi_N|\Psi\rangle, N_S = \langle\Psi|\Pi_S|\Psi\rangle.$$

Figure 5 shows the time dependence of N_W in the controls without CDF (dashed curve) and with CDF (solid curve). The figure clearly shows that the nonadiabatic transitions at $t \sim T/2$ are greatly suppressed and transitions take place only at around $t = T/2$. This means that our approximation introduced in the denominator of (7) is not justifiable at $t \sim T/2$, where the original \mathbf{B}_{CD} diverges at $r = 0$. Otherwise, our simple approximation

works reasonably well.

Now an important observation is in order. In our original proposal, we need to prepare two sets of Ioffe bars to produce the original confining quadrupole field and the counter-diabatic magnetic field. Clearly this is demanding for experimentalists. All topological vortex formation experiments so far were conducted with a single set of Ioffe bars. Even if one could build a trap with two sets of Ioffe bars, aligning their centers exactly at the same place is practically impossible. To circumvent this problem, we introduce time-dependent gauge transformation so that the combined field $\mathbf{B}_\perp + \mathbf{B}_{\text{CD}}$ is rotated and the resulting field is in parallel to the original \mathbf{B}_\perp . Let

$$\alpha_B = \tan^{-1} \frac{|\tilde{\mathbf{B}}|}{|\mathbf{B}_\perp|} \quad (10)$$

be the angle between $\tilde{\mathbf{B}}$ and \mathbf{B}_\perp . If $\tilde{\mathbf{B}}$ is rotated by $-\alpha_B$, it becomes parallel to \mathbf{B}_\perp and hence it can be generated by a single set of Ioffe bars. This rotation is implemented by the unitary transformation

$$U(\alpha_B) = e^{i\alpha_B F_z}. \quad (11)$$

Now the Zeeman term of the Hamiltonian is transformed as

$$U(\alpha_B)\gamma\tilde{\mathbf{B}} \cdot \mathbf{F}U^\dagger(\alpha_B) = \gamma\tilde{\mathbf{B}}_\perp \cdot \mathbf{F}, \quad (12)$$

where $\tilde{\mathbf{B}}_\perp = |\tilde{\mathbf{B}}|(x, -y, 0)$. Of course, this is not the whole story and there is a price we need to pay. The Hamiltonian in the rotating frame acquires a gauge term

$$-iU\partial_t U^\dagger = -\dot{\alpha}_B F_z. \quad (13)$$

This term, proportional to F_z , works as a magnetic field in the z -direction and the bias field $B_z(t) = B_z(0)(1 - 2t/T)$ is replaced by

$$\tilde{B}_z(t) = B_z(0) \left(1 - \frac{2t}{T}\right) - \dot{\alpha}_B. \quad (14)$$

Note that α_B depends on time but not on space coordinates. Numerical calculation shows that the number of atoms in WFSS in the control with $\tilde{\mathbf{B}}_\perp$ and \tilde{B}_z is exactly the same as that for two sets of Ioffe bars for $t \geq T$. Figure 6 shows $\tilde{B}_z(t)$ and $|\tilde{\mathbf{B}}_\perp(t)|/|\mathbf{B}_\perp|$ for ^{23}Na with the parameters used in Fig. 5.

In summary, we have proposed a method to suppress nonadiabatic transitions while topological vortex formation takes place in BEC of alkali atoms. The counter-diabatic field is generated by a set of Ioffe bars, which is obtained by rotating the original Ioffe bars producing the confining quadrupole field by $\pi/4$. Our numerical calculation demonstrates that nonadiabatic transitions are suppressed for any inversion time T and, in particular, it is most impressive for a small T . We can further improve this scheme by applying a gauge transformation to a rotating frame so that the combined field $\tilde{\mathbf{B}}$ is parallel to

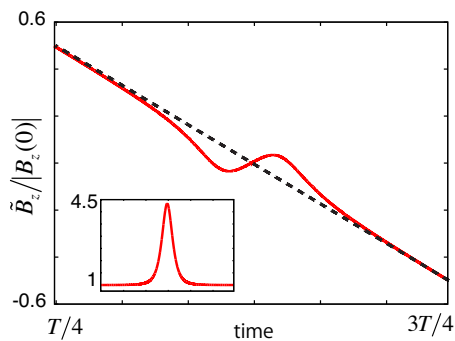


FIG. 6. (Color online) Time dependence of $\tilde{B}_z(t)/B_z(0)$ (solid curve) and $B_z(t)$ (dashed line) for the same parameters used in Fig. 5. The vertical and horizontal axes of the inset are $|\tilde{B}_\perp(\mathbf{r}, t)|/|B_\perp(\mathbf{r})|$ and time with the range $T/4 \leq t \leq 3T/4$, respectively.

B_\perp . This requires modulation of $B_z(t)$ from linear time-dependence. We believe our proposal is experimentally feasible by simple modifications of the existing setup.

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